THE EFFECT OF LIQUID SPREAD ON THE TOP OF THE BED ON RADIAL SPREAD WITHIN THE PACKING*

V. STANĚK

Institute of Chemical Process Fundamentals,
Czechoslovak Academy of Sciences, 165 02 Prague 6 - Suchdol

Received February 16th, 1976

It is shown in the paper that the experimentally observed increase of the coefficient of radial spread, D, with the mean density of irrigation in randomly packed trickle-flow beds can be quantitatively accounted for by the spread of liquid on the top of the bed. With the aid of the Ergun equation the effect of the spreading on the top can be computationally eliminated, this being simultaneously accompanied with a substantial improvement of the agreement of experimental values of D obtained on beds of different depth. The resulting D's then, on the contrary, display a decreasing trend with increasing mean density of irrigation this trend being substantially weaker in case of Raschig rings in comparison with a bed of spheres. The computational elimination of the effect of liquid spread on the top of the bed appears more difficult for Raschig rings due to their spatial orientation.

The manner in which liquid spreads radially within a bed of packing under the free trickle flow can be assessed experimentally most conveniently from the profiles of the density of irrigation established within the bed after irrigating the top by a central "point" jet of liquid. In some processes central jet is used also in practice despite of the fact that from the viewpoint of mass transfer is not favourable. The reason for industrial use of jet liquid distributors is their construction simplicity, low rate of failure in comparison with rotating distributors and large area of cross section available for gas flow.

From the standpoint of theoretical studies of radical spread of liquid the "point" distributor appears attractive mainly because of the relative simplicity of solution of the appropriate differential equation[1] governing the distribution of liquid in random packings. In addition, if the depth of the examined bed is not great, no measurable amount of liquid reaches the wall of the column. The boundary condition, describing the differential behaviours of liquid in the proximity of the wall, then introduces no additional parameter into the solution and the problem of evaluating the coefficient of radial spread thus reduces to a one-parameter search.

* Part X in the series Distribution of Liquid over a Random Packing; Part IX: This Journal *39*, 2007 (1974).

For shallow beds the packing can be regarded with sufficient accuracy as unlimited and thus[1,2]

$$f/f_0 = 1/(4Z) \exp\left[-r^{*2}/(4Z)\right]. \tag{1}$$

Alternatively, the wall can be regarded as a perfect sink[1] for liquid and the packing in the proximity of the wall remains perfectly "dry":

$$f/f_0 = \sum_n \frac{J_0(q_n r^*)}{J(q_n)} \exp\left(-q_n^2 Z\right). \tag{2}$$

The eigenvalues q_n are given by the equation

$$J_0(q_n) = 0\,. \tag{3}$$

Similarly the wall can be viewed as a perfect reflector for liquid[1], *etc.*

None of these solutions fits the distribution of liquid in deeper beds but it can be shown that for shallow beds the resulting theoretical profiles are similar.

The coefficient of radial spread contained in the dimensionless height Z can be evaluated from comparison of experimental values with the theoretical ones for various Z until a satisfactory agreement is reached. One can compare *e.g.* directly the profiles of density of irrigation or some derivative quantities such as flow rates through an annulus, *etc.* However, regardless of the method of evaluation, the results, expressed in terms of the coefficient D corresponding to various f_0, display considerably increasing trend with growing mean density of irrigation and depend also on the depth of the bed (Fig. 5*a*). Because the amount of liquid that must be brought into the central point source, represented in practice by small-diameter jet, are large it is logical that the increase of D with growing f_0 has been ascribed to the flooding of the interstices of the packing at the top near the axis.

In this paper it will be shown that the main cause for the apparent increase of D with the mean density of irrigation is the spread of liquid on the top of the bed eventually within a shell below the top surface of the bed.

THEORETICAL

The liquid discharged from a jet encounters the top surface of the bed initially on a small area given by the cross section of the jet or even smaller due to the contraction of the jet at low discharge velocities (Fig. 1). Individual peces of packing occupy a certain part of the top cross sectional area leaving the free cross section in the form of openings of irregular shape. The characteristic size of these openings related to the size of a piece of packing (d_p). The number of these openings directly hit by the discharged jet of liquid is small and the openings do not suffice to absorb all liquid. If all liquid were to be absorbed only by the directly hit openings the required static head above these openings would be very high. Instead, the liquid actually spreads radially already at the top of the bed. Typical outline of the liquid spreading over the top of the bed is shown in Fig. 1 as curve 4. As a consequence of the spreading, additional openings in the top cross section become accessible to the liquid and this process continues so long as the height of the liquid on the top surface drops to a practically small values.

For purpose of describing this state mathematically we shall assume that the layer of liquid on the top surface of the bed does not vary in radial direction. The pressure losses of liquid flowing through a porous medium may be expressed by means of the Ergun equation[3] as

$$\Delta P/z = 150\mu\, v_0(1 - \varepsilon)^2/d_p^2\varepsilon^3 + 1{\cdot}75\varrho\, v_0(1 - \varepsilon)/d_p\varepsilon^3 \,. \tag{4}$$

In our case though the pressure force is due to the weight of liquid which is expressed by replacing the left hand side of the last equation by the specific weight of the liquid:

$$\varrho \boldsymbol{g} = 150\mu\, v_0(1 - \varepsilon)^2/d_p^2\varepsilon^3 + 1{\cdot}75\varrho\, v_0(1 - \varepsilon)/d_p\varepsilon^3 \,. \tag{5}$$

Superficial velocity of liquid related to the area hit by the liquid on the top surface of the bed, v_0, appearing in Eq. (5) can be expressed in terms of the mean density of irrigation (actually the superficial velocity of liquid related to the whole cross section of the column) and the radius of the disc, r_j, of liquid on the top of the bed fed by the jet as:

$$f_0\pi R^2 = v_0\pi r_j^2 \,. \tag{6a}$$

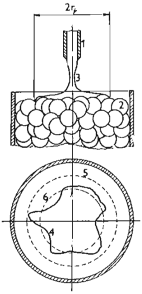


FIG. 1

Cut-Away View of the Bed of Spheres and the Contour of Liquid Spreading on the Top of the Bed

1 Nozzle; 2 bed of spheres; 3 liquid; 4 contour of spreading liquid; 5 smallest circle encompassing all liquid spreading on the top; 6 circle of area equivalent to the area covered by the spreading liquid.

In the dimensionless form then

$$r_j^{*2} = f_0/v_0 \, . \tag{6b}$$

Substituting this quantity into the Ergun equation in the form (5) a quadratic equation for the quantity (r_j^{*2}/f_0) is obtained

$$(r_j^{*2}/f_0)^2 - A(r_j^{*2}/f_0) - B = 0 \, , \tag{7}$$

where

$$A = 150\mu(1 - \varepsilon)^2/(\varrho g d_p^2 \varepsilon^3) \, , \tag{8a}$$

$$B = 1{\cdot}75(1 - \varepsilon)/(d_p \varepsilon^3 g) \, . \tag{8b}$$

Using this simplified model the radius of the disc of liquid appearing on the top of the packing fed by the jet of a small diameter can be expressed by

$$r_j^* = k(f_0)^{1/2} \, . \tag{9a}$$

The value of the constant k follows from the parameters A and B of the quadratic equation (7) and depends only on physical properties of the liquid, the size and the shape of a piece of packing and the porosity of the bed. Because A^2 is mostly much smaller then B, the constant k may be expressed with sufficient accuracy only in terms of B as

$$r_j^* = B^{1/4} f_0^{1/2} \, . \tag{9b}$$

Provided that no measurable amount of liquid reaches the wall the distribution of the density of irrigation in a packing irrigated by a small-diameter central jet should obey the theoretical distribution valid for a disc distributor[1]

$$f/f_0 = (2/r_1^*) \sum_n \frac{J_1(q_n r_1^*) \, J_0(q_n r^*)}{q_n \, J_1^2(q_n)} \exp\left(-q_n^2 Z\right) . \tag{10}$$

In this expression we substitute for the radius of the disc, r_1^*, the radius of the disc of liquid on the top surface of the bed. Namely, r_j^*.

EXPERIMENTAL

The radius of the disc of liquid on the top of the bed fed by the jet was determined experimentally in several ways:

1. *Photographic method*: By photographing the top surface of the bed irrigated by a coloured liquid. The coloured liquid used for experiments was a solution of potassium permanganate fed

into the jet. The photographs of the top surface, taken by parts (one quadrant at a time), were enlarged and the outline between the dark and light surfaces traced on a sheet of paper. The area of the dark surfaces was evaluated by a planimeter and expressed by equivalent diameter of a disc. The equivalent diameter of the disc is shown schematically in Fig. 1 as the circle 6.

2. *Extrapolation method*: By observing the distribution of liquid on a very shallow bed of packing irrigated by central jet. The height of the bed was gradually increased and the obtained fractional flow rates of liquid passing through a set of concentric cylinders below the bed were plotted as functions of bed depth and extrapolated to zero depth. The extrapolated values of fractional flow rates of liquid were in turn plotted against the radius of the appropriate collecting cylinder in order to find out the radius for which this curve intersects with the abcissa passing through unity on the axis of fractional flow rates. In other words, we found the minimum radius encompassing all liquid on the top of the packing. This minimum radius is shown in schematically in Fig. 1 as circle 5.

3. *Method of multiparameter regression*: By calculating the radius of the disc of liquid on the top of the bed by multiparameter regression of the data on distribution of liquid from central jet. The optimized parameters of this regression were the coefficient of radial spread D and the radius of the disc of liquid on the top of the bed.

The regression proceeded in such a manner that for a preselected value of the hypothetical disc distributor the optimum value of the dimensionless depth of the bed, Z, was found corresponding to the least sum of square deviations of the measured and calculated profiles of the density of irrigation. Next a new radius of the hypothetical disc was taken and the optimization of Z repeated. The strategy of selecting the new values for the radius of the hypothetical disc was such as to minimize the sum of square deviations also with respect to this quantity.

The apparatus used for the measurement of the distribution profiles has been described earlier[2]. The method rests in principle in separating the liquid draining from the packing by means of a set of concentric cylinders and clocking the time required to fill vessels of known volume. This primary data were recalculated to yield the density of irrigation profiles.

The inner diameter of the employed cylindrical glass column was 291 mm and the measurements included beds of spheres 15 and 20 mm in diameter and 15 and 25 mm Raschig rings. The irrigating liquid was in all cases water.

RESULTS AND DISCUSSION

The results in the form of dimensionless radius of the disc of liquid fed by liquid jet on the top of the bed are shown in Fig. 2. The results were obtained from measurements on a bed of 20 mm spheres. The curve *1* was computed from the Ergun equation written in the form (7). The values obtained by the photographic method are mostly higher and were correlated by Eq. (*9a*) following from the functional relationship given by the Ergun equation. However, the value of the parameter k was optimized with respect to the experimental data. The curve *3* also satisfies Eq. (*9a*) but the parameter was optimized with respect to the data obtained by the extrapolation method.

The difference between the measurements given by the curves *2* and *3* stems from the fact that the extrapolation method yields the minimum radius encompassing all liquid on the top while the photographic method yields the equivalent diameter of the disc of liquid. The photographic method thus necessarily provides lower values because the areas covered by liquid on the top of the bed are strongly noncircular.

Fig. 3 illustrates the course of the standard deviation of the experimental profiles of the density of irrigation obtained from measurements with the central jet on the bed of 20 mm spheres 300, 500 and 875 mm deep compared with the theoretical profiles for disc distributors of various diameters (Eq. (*10*)). The criterion for comparison was to find such a value of Z for which the measured and computed profiles exhibited for the given radius of the disc distributor the least standard deviation. The vertical lines indicate the value of the dimensionless radius of the disc computed from the Ergun equation, obtained by the photographic and the extrapolation method. From the figure it is seen that the optimum radii of the disc for the three depths of the bed are relatively close to each other and well agree with the value obtained by extrapolation on shallow beds.

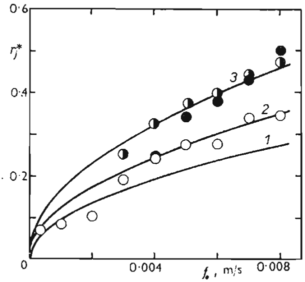


FIG. 2

Equivalent Radius of the Disc of Spreading Liquid as a Function of the Mean Density of Irrigation

Curves correspond to the functional dependence $r_j^* = kf_0^{1/2}$ with k obtained by: *1* calculation from the parameters of the Ergun equation; *2* photographic method; *3* extrapolation method. Extrapolation points were obtained by: ○ photographic method; ● extrapolation method; ◐ multiparameter regression.

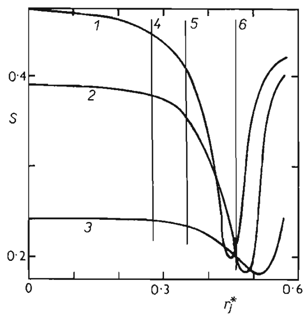


FIG. 3

Course of Standard Deviation of the Experimental Profiles of the Density of Irrigation for 20 mm Spheres at $f_0 = 0{\cdot}008$ m/s Irrigated by Central Jet

Experimental profiles were compared with the theoretical profiles corresponding to a disc distributor of radius r_j^*; *1* $z =$ = 300 mm; *2* 500 mm; *3* 875 mm; *4* r_j^* computed from the Ergun equation; *5* r_j^* corresponding to the results of the photographic method; *6* r_j^* corresponding to the results of the extrapolation method.

Fig. 4 illustrates the regression method listed as the third method used to evaluate the radius of the disc of liquid on the top of the bed fed by the jet. However, instead of taking individual standard deviation *versus* r_j^* curves we took a single standard deviation representing the deviation of the measurements on all three depths of the bed.

Curve *1* indicates the course of the standard deviation of measurement with Raschig rings on 300 and 500 mm deep beds when the theoretical profiles were computed with the optimum Z individually for each depth. Curves 2 and 3, on the contrary, represent the coefficients of radial spread computed from the optimum Z's in dependence on the value of the dimensionless radius of the hypothetical disc distributor. Both measurements were carried out at the mean density of irrigation 0·003 m/s. What is characteristic in comparison with the previous figure is that with decreasing mean density of irrigation, and hence with decreasing mean diameter of the hypothetical disc distributor, the local minimum of the standard deviation becomes less conspicious. It is so because for low radii of the disc distributor the high local gradients of density of irrigation near the axis of the column quickly fade and the problem becomes little sensitive to the radius r_j^*. This also shows on Fig. 2 from which it is apparent that r_j^* found for f_0 less than about 0·003 m/s display a different trend.

From Fig. 4 it can be further seen that for the given case the local minimum of the mean standard deviation almost agrees with the intersect of the optimum values of D for the measurements on the two depths of the bed. The agreement of these values is a necessity, of course, if D is to be a parameter independent of the density of irrigation. This, as shall be shown later, is not perfectly true. Nevertheless, it seems that the agreement of D's measured on beds of various depth, or the minimum deviation of the optimum values of D in case of measurements on several depths

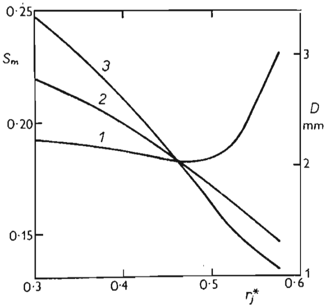


FIG. 4

Mean Standard Deviation of Experimental Profiles of the Density of Irrigation for 25 mm Raschig Rings at $f_0 = 0·003$ m/s Irrigated by Central Jet

1 Course of mean standard deviation of measurements on 300 and 500 mm deep beds and optimum values of the dimensionless depth Z; the theoretical profiles were computed for the disc distributor of radius r_j^*; *2* course of optimum D following from measurement on 300 mm deep bed; *3* optimum D for 500 mm deep bed.

of bed may serve as a good criterion for determining optimum r_j^*, particularly for low f_0 when, as already mentioned, the minimum standard deviation is a poorly sensitive criterion.

The values of the parameter k defined in Eq. (*9a*) calculated by various procedures for a bed of 20 mm spheres are summarized in Table I. From the table it is apparent that the value calculated directly from the Ergun equation for the given packing and the mean porosity $\varepsilon = 0{\cdot}4$ typical for spheres is lower than the values obtained by any other method used. The discrepancy can be explained as follows: The above value of porosity represents a mean value characteristic of the whole bulk of the packing. In the proximity of containing surfaces such as *e.g.* walls of the column, however, the porosity varies approximately sinusoidally while the amplitude of the fluctuations deminishes with increasing distance from the containing surface[4-8]. The top surface of the bed though is not a fixed surface but due to the levelling of the top of the bed after filling the column the conditions here are quite similar to those near a plane surface: In the level of the top surface of the bed the porosity equals unity and this value diminishes in vertical direction into the bed. The course of the porosity near a confining plane has been experimentally studied by Benenati and Brosilow[5]. According to their results the packing exhibits minimum porosity at $d_p/2$ from the confining plane and the minimum porosity equals approximately 0·2. The value of the parameter k computed from the Ergun equation for this minimum value is also shown in Table I and appears to be in remarkable agreement with the values obtained by multiparameter regression and extrapolation.

Thus it is likely that the radius of the disc of liquid on the top of the packing fed by a jet is not dictated by the mean porosity of the bed but rather by the minimum value existing just below the top surface ($d_p/2$). The ultimate result of this is an increased effective value of the parameter k.

The finding regarding the spread of liquid on the top surface of the bed were further utilized to evaluate the coefficient of radial spread within the bed, D.

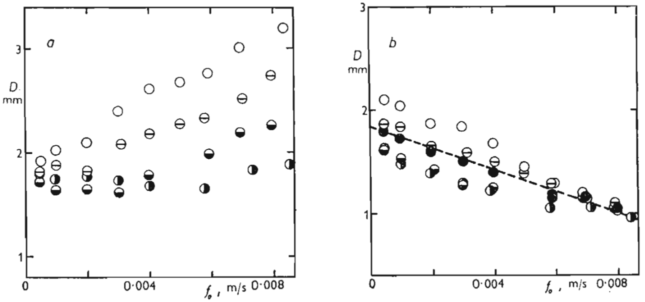
TABLE I

Values of Parameter k of the Dependence $r_j^* = kf_0^{1/2}$ for 20 mm Spheres

Method of evaluation of k	k, $(s/m)^{1/2}$
Calculated from the Ergun equation ($\varepsilon = 0{\cdot}4$)	3·04
Photographic method	3·85
Extrapolation	5·12
Multiparameter regression	5·16
Calculated from the Ergun equation ($\varepsilon = 0{\cdot}2$)	5·46

Fig. 5*a* shows the results of the coefficient of radial spread obtained with 15 mm spheres on four different depths of the bed. The bed was irrigated by central jet and the obtained profiles of the density of wetting were compared with the theoretical distribution curves computed from Eq. (*2*). The computed profiles were compared for different values of the dimensionless height Z until a minimum deviation in terms of the sum of mean squares was reached. This optimum Z served to compute D. the obtained D's increase with the mean density of irrigation and the results obtained on beds of different depth are considerably different, these differences being particularly conspicuous for higher mean densities of irrigation.

Fig. 5*b* plots the results obtained from the same experimental data as those in the



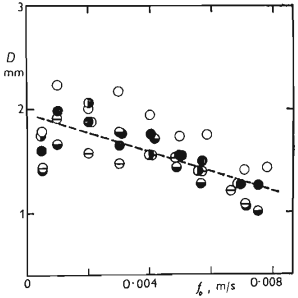
a b

FIG. 5

Coefficient of Radial Spread of Liquid for 15 mm Spheres as a Function of Mean Density of Irrigation and Central Jet

a) ○ z = 200 mm; ⊖ 300 mm; ◒ 400 mm; ◑ 570 mm.

b) Theoretical profiles were computed from the solution for a disc distributor and r_j^* obtained by the photographic method. ○ z = 200 mm; ⊖ 300 mm; ◒ 400 mm; ◑ 570 mm; ● mean values weighted by the standard deviation of the profiles. Broken line indicates approximate course of the dependence.



c

c) Coefficient of Radial Spread of Liquid for 15 mm Spheres as a Function of Irrigation and Central Jet with Diameter $r_1 = 1/2$. ○ z = 100 mm[1]; ⊖ 300 mm; ◒ 400 mm; ◐ 570 mm; ● mean values weighted by the profiles. Broken line indicates approximate course of the dependence.

preceding figure but the theoretical profiles used for evaluating D were computed from the solution for a disc distributor written in Eq. (*10*). The radius of the disc of liquid on the top of the bed was computed for the given mean density of irrigation from the Ergun equation (*9a*) with an empirical coefficient, k, corrected in accord with the results of the photographic method. From the figure it may be apparent that D's corresponding to beds of different depth now differ only little. On the other hand, the dependence of D did not disappear but took an opposite trend. This trend now is very similar to that displayed by the results of D obtained on the same packing but irrigated by a disc distributor of radius $r_1 = 1/2$. The results with this real disc distributor are shown in Fig. 5*c*.

Similar conclusions can be drawn also for 20 mm spheres. In case of Raschig rings the evaluation based on the concept of the disc of liquid on the top of the bed also virtually removes the differences between the results measured on beds of different depth. However, the resulting new trend of D with f_0 is weak.

CONCLUSION

The results of measurement of the distribution of the density of irrigation from a central jet were processed as profiles produced by a hypothetical disc distributor whose radius was computed from the Ergun equation. It turned out that with the empirical coefficient of the Ergun equation corrected at least in accord with the results of photographing the liquid spreading over the top of the bed the agreement of D's obtained on beds of different depth considerably improves. The radii of the disc on the top of the bed obtained by extrapolation or multiparameter regression, however, are higher than those of the former method. This also seems to confirm the idea that spreading atypical for the bed proper does not take place on the very top of the bed only but within a certain depth ($d_p/2$) below the top surface.

After processing the data assuming a disc of liquid on the top of the bed the originally strong increasing trend of D with the mean density of irrigation changed into a decreasing one, this being in good agreement with the course of the results obtained with the real disc distributor of radius $r_1 = 1/2$. Obviously, if there is a certain correlation between D and local density of irrigation both $D - f_0$ courses can in principle be identical only if the initial distribution is also identical.

These results indicate that direct evaluation of the coefficient of radial spread, D, from the profiles of the density of irrigation using a jet is virtually ruined by the peculiarities of the spreading on the very top of the bed where the liquid is being fed on a relatively small area. As has been shown the results can be successfully evaluated if it is considered that the original central jet is transformed on contact with the packing into a hypothetical disc distributor whose radius can be predicted in dependence on the mean density of irrigation. However, in case of Raschig rings and the majority of practically used packings lacking spherical symmetry the prediction of the radius of the disc is complicated by the fact that local resistance of *e.g.* Raschig rings depends on their spatial orientation. The rings composing the top of the bed are, moreover, mostly oriented with their axis horizontally as a consequence of levelling the top of the bed after filling. For a given radius of the column the adverse effect of this peculiar section of the bed on the results of evaluation will increase with the growing size of the rings because a single layer of the packing (d_p) represents ever greater portion of the whole depth of the bed.

As a general approach for measuring the spreading of liquid in randomly packed columns one can reccmmend the approach consisting essentially from measurements on two beds of identical particles but different depth. The greater depth of these two, say z_1, is chosen so as to satisfy the appropriate boundary condition (*e.g.* "dry" wall), the latter depth, say z_2, is taken very small and equal for instance $z_2 = 3d_p$. The found profile of the density of irrigation for z_2 then serves as the initial condition for evaluating D from the measurements on z_1. Naturally, the resulting D's correspond to the depth $z_3 = z_1 - z_2$. If z_2 is sufficiently small compared to z_3 the data processing may be simplified by replacing the general profile corresponding to z_2 by a profile simulating a disc distributor of equivalent diameter. The evaluation can then be done with the aid of analytical solutions. If z_2 is not sufficiently small to make this approximation justified the profile can be approximated by a function which is the analytical solution of the given problem for the used boundary condition and the following procedure is again simple.

LIST OF SYMBOLS

A, B	parameters defined by Eqs (*8a*, *b*)
D	coefficient of radial spread of liquid, (L)
d_p	characteristic particle size, (L)
f	density of irrigation, (LT^{-1})
f_0	mean density of irrigation, (LT^{-1})
g	acceleration due to gravity, (LT^{-2})
J_0, J_1	Bessel functions of the first kind, zero and first order
k	parameter defined by Eq. (*9a*)
n	summation index
ΔP	pressure drop, ($ML^{-1}T^{-2}$
q_n	eigenvalues defined by Eq. (*3*)
R	radius of cylindrical column, (L)
r	radial coordinate, (L)
$r^* = r/R$	dimensionless radial coordinate
r_j	radius of disc of liquid on the top of bed fed by central jet, (L)
$r_j^* = r_j/R$	dimensionless radius of disc of liquid on the top of bed fed by central jet
r_1	radius of disc distributor, (L)
$r_1^* = r_1/R$	dimensionless radius of disc distributor

S standard deviation
S_m mean standard deviation
v_0 superficial velocity of liquid related to the area of top column cross section covered by liquid, (LT^{-1})
$Z = Dz/R^2$ dimensionless depth of bed
z coordinate of height, (L)
z_1, z_2, z_3 depth of bed, (L)
μ viscosity of liquid $(ML^{-1}T^{-1})$
ε porosity of bed
ϱ density of liquid (ML^{-3})

REFERENCE

1. Staněk V., Kolář V.: This Journal *33*, 1049 (1967).
2. Staněk V., Kolář V.: This Journal *38*, 1012 (1973).
3. Ergun S.: Chem. Eng. Progr. *48*, 89 (1952).
4. Roblee L. H. S., Baird R. M., Tierney J. W.: AIChEJ *4*, 460 (1958).
5. Benenati R. F., Brosilow C. B.: AIChE J. *8*, 359 (1962).
6. Thadani M. C., Peebles F. N.: Ind. Eng. Chem., Process Des. Develop. *5*, 265 (1966).
7. Kondelík P., Horák J., Tesařová J.: Ind. Eng. Chem., Process Des. Develop. *7*, 250 (1968).
8. Kondelík P., Boyarinov A. I.: This Journal *34*, 3853 (1969).

Translated by the author.